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$$\frac{dy}{dx}=\frac{r-2x}{2\sqrt{(rx-x^2)}},\qquad \frac{dz^2}{dx^2}=1+\left(\frac{dy}{dx}\right)^2=\frac{r^2}{4(rx-x^2)},\qquad z^2=r^2-x^2-y^2=r^2-rx.$$

$$\therefore\ L=2r\sqrt{r}\int_0^r\frac{dx}{\sqrt{x}}=4r^2.$$

The spherical surface removed, $S=4\int_0^r dx\int_0^{\sqrt{(rx-x^2)}}dy\sqrt{1+\left(\frac{dz}{dx}\right)^2+\left(\frac{dz}{dy}\right)^2}$

$$=4r\int_0^r dx\int_0^{\sqrt{(rx-x^2)}}\frac{dy}{\sqrt{(r^2-x^2-y^2)}}=4r\int_0^r\sin^{-1}\sqrt{\frac{x}{r+x}}.dx=(\pi-2)2r^2.$$

Note. **The results of Problem 6 in Byerly's *Integral Calculus*, page 282, 2nd Ed. (1898) are correct, if *b* represents the diameter of the cylindrical hole instead of the radius.**

Also solved by *H. C. WHITAKER.*

113. Proposed by JOHN M. COLAW, A. M., Monterey, Va.

At what rate per unit of time are the roots of the equation ($x^2+px+q=0$) changing, if $p=mq$ and q varies uniformly at the rate of 1/12 per unit of time, when $p=12$ and m remains constant?

Solution by H. C. WHITAKER, A. M., Ph. D., Manual Training School, Philadelphia, Pa.; J. SCHEFFER, A. M., Hagerstown, Md.; and G. B. M. ZERR, A. M., Ph. D., The Temple College, Philadelphia, Pa.

The roots of the given equation are

$$x=\tfrac{1}{2}[-mq\pm\sqrt{(m^2q^2-4q)}],$$
$$dx=\tfrac{1}{2}[-m\pm(m^2q-2)(m^2q^2-4q)^{-\frac{1}{2}}]dq.$$

When $q=12\div m$ and $dq=\frac{1}{12}$,

$$dx=\tfrac{1}{24}\left[-m\pm\frac{(6m-1)\sqrt{m}}{2\sqrt{(9m-3)}}\right].$$

Also solved by *WILLIAM HOOVER.*

114. Proposed by JOHN M. COLAW, A. M., Monterey, Va.

If two concentric ellipses have equal axes inclined at an angle ω, their common area is

$$A=2ab\tan^{-1}\left(\frac{2ab}{(a^2-b^2)\sin\omega}\right).$$

Solution by J. SCHEFFER, A. M., Hagerstown, Md., and J. B. GREGG, Senecaville, Ohio.

Let AB be the semi-major axis of one ellipse, and AB' that of the other, MAC and NAD being common diameters. F is the intersection of AB with the second ellipse, and E and G are the intersections of major axis of the second ellipse with the first.

Let angle $B'AB=\omega$. It can be easily shown that angle $CAF=\frac{1}{2}\omega$, and angle DAF=angle $DAG=\frac{1}{2}\pi-\frac{1}{2}\omega$.

The polar equation of an ellipse referred to the center as pole is

$$r^2=\frac{a^2b^2}{a^2\sin^2\theta+b^2\cos^2\theta}.$$

Since section $EAC=CAF$, and $DAF=DA'G$; we have for $ECFA$ integral

$$a^2b^2\int_{\frac{1}{2}\omega}^{\omega}\frac{d\theta}{a^2\sin^2\theta+b^2\cos^2\theta}=ab\left[\tan^{-1}\left(\frac{a}{b}\tan\omega\right)-\tan^{-1}\left(\frac{a}{b}\tan\tfrac{1}{2}\omega\right)\right]$$

$$=ab\tan^{-1}\left(\frac{ab\sin\omega}{a^2\sin^2\omega+b^2\cos^2\omega+b^2\cos\omega}\right).$$

To find the area of $AFDG$ we have only to put $\pi-\omega$ in place of ω, and thus we get

$$ab\tan^{-1}\left(\frac{ab\sin\omega}{a^2\sin^2\omega+b^2\cos^2\omega-b^2\cos\omega}\right).$$

$\therefore$ Area of $NCDM$

$$=2ab\left[\tan^{-1}\left(\frac{ab\sin\omega}{a^2\sin^2\omega+b^2\cos^2\omega+b^2\cos\omega}\right)+\tan^{-1}\left(\frac{ab\sin\omega}{a^2\sin^2\omega+b^2\cos^2\omega-b^2\cos\omega}\right)\right]$$

$$=2ab\tan^{-1}\left[\frac{2ab}{(a^2-b^2)\sin\omega}\right].$$

Also solved by *G. B. M. ZERR.*

115. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, Ohio.

The axes of two right elliptic cylinders intersect at right angles in such a manner that the *major axes of the sections* are perpendicular. Supposing the axes to be $(A,\ B)>(a,\ b)$, what is the common volume ?

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let $x^2/B^2+z^2/A^2=1$, $x^2/a^2+y^2/b^2=1$, be the equations to the cylinders.

$$\therefore\ V=\frac{8Ab}{aB}\int_0^B\sqrt{[(a^2-x^2)(B^2-x^2)]}dx,\ B<a\ldots\ (1).$$

$$V=\frac{8Ab}{aB}\int_0^a\sqrt{[(B^2-x^2)(a^2-x^2)]}dx,\ a<B\ldots.(2).$$

Let $x=B\sin\theta$ in (1).

Then $V=8ABb\int_0^{\frac{1}{2}\pi}\sqrt{[1-(B^2/a^2)\sin^2\theta]}\cos^2\theta d\theta$